

# Mathematics II

(English course)

Second semester, 2012/2013

## Exercises (9)

1. Find the general solutions for the following differential equations.

(a)  $xy' - y = y^3$ .

(b)  $xyy' = 1 - x^2$ .

(c)  $y - xy' = a(1 + x^2y')$ .

(d)  $3e^x \operatorname{tg} y dx + (1 - e^x) \sec^2 y dy = 0$ .

(e)  $y' \operatorname{tg} x = y$ .

2. Solve the following Cauchy problems:

(a)  $(1 + e^x) yy' = e^x$ ,  $y(0) = 1$ .

(b)  $(xy^2 + x) dx + (x^2y - y) dy = 0$ ,  $y(0) = 1$ .

(c)  $y' \sin x = y \ln y$ ,  $y\left(\frac{\pi}{2}\right) = 1$ .

3. Find the general solutions for the following differential equations.

(a)  $y' = \frac{y}{x} - 1$ .

(b)  $\frac{dy}{dx} - \frac{y}{x} = x$ .

(c)  $\frac{dy}{dx} + \frac{2y}{x} = x^3$ .

4. Solve the following Cauchy problems:

(a)  $xy' + y - e^x = 0$ ,  $y(a) = b$ .

(b)  $y' - \frac{y}{1-x^2} - 1 - x = 0$ ,  $y(0) = 0$ .

(c)  $y' - y \operatorname{tg} x = \frac{1}{\cos x}$ ,  $y(0) = 0$ .

5. Find the general solutions for the following differential equations.

(a)  $y^2 dx + (2xy + 3) dy = 0$ .

(b)  $\frac{dy}{dx} + \frac{y}{x} = -xy^2$ .

(c)  $2xy \frac{dy}{dx} - y^2 + x = 0$ .

(d)  $y dx + \left(x - \frac{x^3 y}{2}\right) dy = 0$ .

(e)  $3xdy = y(1 + x \sin x - 3y^3 \sin x) dx$ .

6. Find the general solutions for the following differential equations.

(a)  $y'' = 1 - y'^2$ .

(b)  $y''y = y'^2$ .

(c)  $(1 + x^2)y'' + y'^2 + 1 = 0$ .

(d)  $x^2y'' + xy' = 1$ .

(e)  $yy'' - y'(1 + y') = 0$ .

(f)  $(x + 1)y'' - (x + 2)y' + x + 2 = 0$ .

(g)  $y'' = -\frac{1}{y^3}$ .

(h)  $xy'' + y' = 0$ .

7. Find the general solutions for the following differential equations.

(a)  $y'' - 5y' + 6y = 0$ .

(b)  $y'' - y' = 0$ .

(c)  $y'' + 2y' + 2y = 0$ .

(d)  $y'' + 2y' + y = 0$ .

(e)  $y'' - ky = 0$ , for  $k \neq 0$ .

(f)  $\frac{y'-y}{y''} = 3$ .

(g)  $y'' - 4y' + 4y = x^2$ .

(h)  $y'' - y' + y = x^3 + 6$ .

(i)  $y'' + 2y' + y = e^{2x}$ .

(j)  $y'' - 8y' + 7y = 14$ .

(k)  $y'' - 4y = x^2 e^{2x}$ .

(l)  $y'' + 9y = \cos 2x$ .

(m)  $y'' - 4y' + 4y = \sin 2x + e^{2x}$ .

(n)  $y'' + 2y' + 2y = e^x \sin x$ .

(o)  $y'' - 5y' + 6y = (x^2 + 1)e^x + xe^{2x}$ .

8. Solve the following second-order Cauchy problems:

(a)  $y'' - 5y' + 4y = 0$ ,  $y(0) = 5$ ,  $y'(0) = 8$ .

(b)  $y'' + 3y' + 2y = 0$ ,  $y(0) = 1$ ,  $y'(0) = -1$ .

(c)  $y'' + 4y = 0, y(0) = 0, y'(0) = 2.$

(d)  $y'' + 2y' = 0, y(0) = 1, y'(0) = 0.$

(e)  $y'' + y' = e^x, y(1) = 0 = y'(1).$

(f)  $y'' + 4y = \sin x, y(0) = 1, y'(0) = -1.$